Mathematical Aptitude and Mathematical Problem-Solving Performance: Three-Wave Panel Analysis

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ABSTRACT

The present study examined the directionality of causal relationship between mathematical aptitude and mathematical problem-solving performance. The sample of this study consisted of 480 eighth grade students drawn through random cluster technique from nine schools of Varanasi, India. Mathematical aptitude test and mathematical problem-solving test were administered on the selected sample at time T_1, T_2 and T_3 with four months time lag. Using cross-lagged panel analytic procedure with three-wave, two-variables (3w2v), symmetric causal relationship was found between mathematical aptitude and mathematical problem-solving performance (i.e., mathematical aptitude precedes mathematical problem-solving performance and vice-versa). The result has been affected due to post hoc fallacy. The investigator can never rule out the existence of third variables that underlie the cross-lag correlations between these two constructs.

Keywords: *mathematical aptitude, mathematical problem-solving performance, cross-lagged panel correlation.*

Mathematics has been recognized as one of the innermost strands of intellectual activity, the foundation of natural sciences, art of wisdom and creativity. The vital role of mathematics is derived from the intellectual, cultural, utilitarian and disciplinary values which the subject seeks to inculcate in the learner. It provides opportunities for opening the mind to new queue of creative ideas. In modern and technological era the knowledge of methods and applications of mathematics has become an integral part of every innovation. Due to this indispensible role mathematics has become an important part of prosperity of a nation. In the interest of student's welfare and country's optimum progress, careful identification and facilitation of mathematical aptitude and mathematical problem-solving performance becomes the most flagrant issues of mathematical education. To use maximum power of young generations for prosperity of a nation, their skills, specific abilities should be identified and continuously guided in appropriate direction to provide adequate training at school level which helps to get a bright career in life. Recently for the facilitation of mathematical aptitude among students, brief autobiography of distinguished mathematicians & Nobel Laureates have

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been included in the text books mathematics of secondary education. It is hoped that it must facilitate mathematical aptitude among students. Bingham (1937) defined that it is a condition symptomatic of a person's relative fitness, of which one essential aspect is his readiness to acquire proficiency – his potential ability – and another is his readiness to develop and interest in exercising that ability (p. 18). Aptitude can be considered as the phase or areas of an individual's mental ability in which he/she can be expected to continue to a point of exceptional performance. However, in the present study mathematical aptitude has been operationally defined as ability to acquire proficiency to perform effectively and precisely on mathematical tasks. In many researches it was asserted that problem solving ability plays an important role in the studying of mathematics. It plays an important role to solve 'real world' problems. Mathematical problem solving is a complex cognitive activity and the heart of mathematics (Halmos, 1980). Polya (1957) defined mathematical problem-solving as a process that involved four dynamic activities: understanding the problem, making a plan, carrying out the plan and looking back. In addition, According to Wallas (1926), humans go through four different stages when trying to solve a problem: preparation, incubation, illumination, and verification. Problem solving involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real world problems.

However, mathematical aptitude has been studied with mathematical creativity, mathematics achievement, skills and other variables. Tuli (1979) reported a significant relationship between mathematics aptitude and mathematical creativity. Burke (1982) reported that mathematical aptitude has a direct effect on college performance, whereas the effect of academic orientation was mediated by high school performance on mathematical aptitude. Gougeon (1984) reported that scholastic aptitude test (SAT) math scores are not a good predictor of college performance, however, it is often used erroneously as exclusive measurements of individual, or institutional, quality. Morris & Bowling (1979) reported that a clear relationship between mathematical aptitude and the discrepancy between confidence and performance. The lower the aptitude leads to the poorer the performance at a given confidence judgment. Researches in the area of mathematical aptitude have reported very important findings but ample evidences have shown a significant relationship among mathematical aptitude and problem-solving performance in mathematics (Tyagi, 2014). But the causal relationship between mathematical aptitude and mathematical problem-solving performance is not yet clearly reported. Therefore, the present study has been conducted to find out the answer of following research questions:

1. Whether mathematical aptitude is the cause of mathematical problem-solving performance or vice-versa?
2. Whether the relationship between mathematical aptitude and mathematical problem-solving performance is symmetrical?
3. Whether the relationship between these two construct is spurious?

Method

Cross-Lagged Panel Analysis (CLPA) technique-a quasi-experimental non equivalent control group design was used to analyses the data which were observed at three points in time.

Sample and Procedure

The sample consisted of 480 eighth standard students included through random cluster technique from nine intermediate and high schools located in Varanasi region, India. Students were involved from both rural and urban locality. The mean age group of the selected sample was 13 years. Finally 480 eighth grade students (83 urban male + 107 urban female + 118 rural male and 172 rural female) participated in three phases (T_1, T_2 & T_3) with four months time lag. Kenny (1975) recommended that complete data must be present for all time periods to undertake cross-lag analysis. The instructions and contents of both the tests were translated from English to Hindi with back-translation procedure to ensure accuracy and equivalency.

Instruments

Mathematical Aptitude Test

Hindi adaptation of Braswell's (1978) mathematical aptitude test developed by the investigator used to measure mathematical aptitude among the middle school students. The test consists of 40 multiple choice items related to the different branches of mathematics i.e., Arithmetic, Algebra, and Geometry and based on knowledge, comprehension and application dimension of Blooms Taxonomy (Bloom, 1956). These items stimulate more on proficiency, preciseness, and effectiveness to solve the given problem. The reliability of the test was found to be .88 (df = 398) by using rational equivalence method. The investigator established the validity of mathematical aptitude test on 160 participants. The item correlations were found to be ranged from .64 to .91. It indicates that mathematical aptitude test possesses item validity.

Mathematical Problem-Solving Performance Test

To measure problem-solving performance in mathematics, the investigator used the Hindi adaptation of Krutetskii's problem solving test developed by Singh (1993). Part – I consists of eight mathematical problems which are related with real life situations. These problems stimulate more on preciseness, analysis and application of mathematical learning competencies for obtaining solutions. Part – II consists of seven problems. Out of these four problems one is of puzzle type and the remaining three are of situational problems. The test-retest reliability of the test was found to be .70. It indicates moderate consistency of the test. Item validity of mathematical problem-solving test was also found to be moderate.

Design and Analysis

A CLPA technique, a quasi-experimental design (Campbell & Stanley, 1963) was independently suggested by Campbell (1963). Although not without its critics (e.g., Rogosa, 1980), it has been used effectively in education research (e.g., Tyagi, 2015, 2016; Tyagi & Singh, 2014; Ahmed et al., 2012; Verma, 1994; & Kenny, 1975). This method utilizes 'panel data', therefore, the rudiments of this analysis necessitates at least two variables which measured at least two times simultaneously.

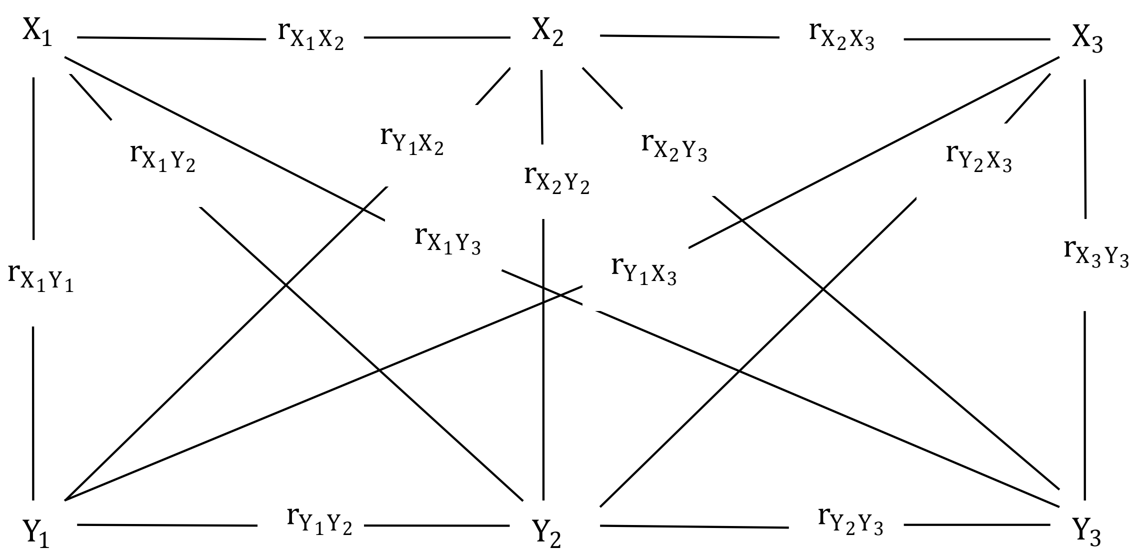

Figure 1. Cross-lagged Panel Correlation Paradigm (X and Y are variables and 1, 2, and 3 are times)

As shown in the Figure 1 that two variables X (mathematical aptitude) and Y (mathematical problem-solving performance) and three lag (time 1, time 2, & time 3) generate six variables (X_1, X_2, X_3, Y_1, Y_2, and Y_3) and six variables generate six correlations: six autocorrelations ($r_{X_1X_2}$, $r_{Y_1Y_2}$; $r_{X_2X_3}$, $r_{Y_2Y_3}$ & $r_{X_1X_3}$, $r_{Y_1Y_3}$); three synchronous correlations ($r_{X_1Y_1}$, $r_{X_3Y_3}$, & $r_{X_2Y_2}$) (cross-section); and six cross-lagged correlations ($r_{X_1Y_2}$, $r_{Y_1X_2}$; $r_{X_2Y_3}$, $r_{Y_2X_3}$; & $r_{X_1Y_3}$, $r_{Y_1X_3}$). CLPA is a method for testing spurious relationships by comparing the cross-lagged differential: $r_{X_1Y_2}$ minus $r_{Y_1X_2}$ ($r_{X_1Y_2} - r_{Y_1X_2}$) and similarly ($r_{X_2Y_3} - r_{Y_2X_3}$) and ($r_{X_1Y_3} - r_{Y_1X_3}$). It is clear that the attribution of causal predominance in CLPC is based on the difference between cross-lagged correlations (and similar process for time-2 & time-3 and time-1 & time-3 respectively. If cross-lagged differential is positive, conclude the causal predominance to be that of *X* causing *Y*, and if the cross-lagged differential is negative, *Y* causing *X*. No significant differences in the cross-lags suggest that the relationship between the variables is spurious or mutually influencing to each other equally. For the effective use of CLPA synchronous correlations should be at least .30 as well as sample size should be large. The null hypothesis of CLPC is that the two variables are not causally related but seem to be affected by some other set of common causes of "third variables" (Simon, 1954). Synchronicity and stationarity are sine qua non for the use of this analysis, (Kenny, 1975). Synchronicity means that the variables involved are measured at the same point in time, a condition which is satisfied in this study. Stationarity, tested by comparing the synchronous correlations means that the causal processes do not change during the time lag measured. No significant differences between the synchronous correlations indicate that variables are stationary i.e., prior relationship exists do not change over times. As shown in Figure 2, the stipulation of stationarity was not satisfied but provided the evidence of the prior relationship between these two constructs, therefore, the cross-lagged correlations and synchronous correlations were corrected to find the reliable and valid results by using the reliability/communality ratios. After correction procedure stipulation of quasi stationarity is satisfied i.e., equal synchronous correlations, otherwise, variables that decrease in reliability would erroneously appear to be causes, while those that increase would erroneously appear to be effects (Campbell, 1963). Pearson-Filon (PF) test was used for testing the difference between the cross-lagged correlations (dependent correlations) because of quasi stationarity exists.

Results

As shown in Figure 2 that unequal synchronous correlations were found between mathematical aptitude and mathematical problem-solving performance, (r_{MA1MP1} = .64, r_{MA2MP2} = .54 and r_{MA3MP3} = .66).

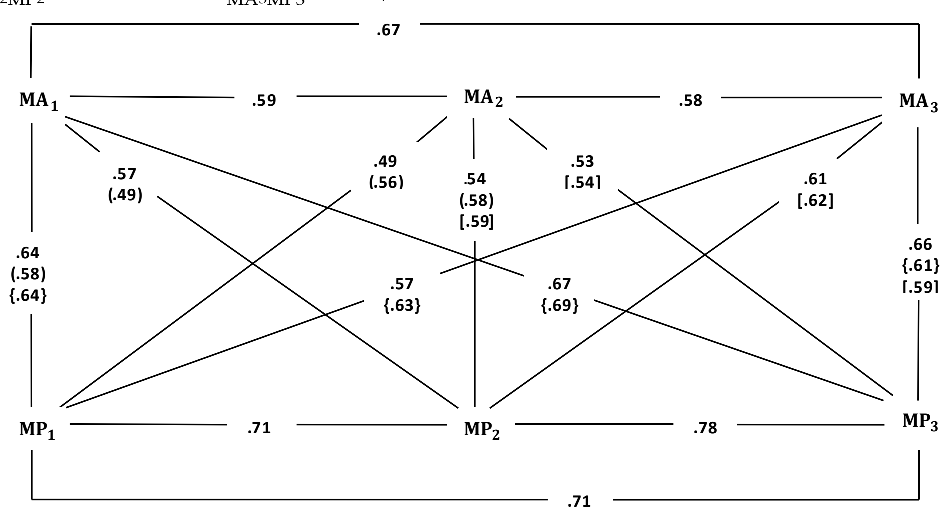

Figure 2. ( ) Correction in synchronous and cross-lagged correlation at time-1 and time-2
[ ] Correction in synchronous and cross-lagged correlation at time-2 and time-3
{ } Correction in synchronous and cross-lagged correlation at time-1 and time-3

Quasi-stationarity exists, a demonstrated by the equality of the synchronous correlation after correction procedure & = (.65), & = {.58}, & = [.64] respectively. However, reliability ratio for mathematical aptitude test was found to be .79, indicating decreasing reliability, whereas the reliability of the mathematical problem-solving performance measure remained essentially unchanged over the two time periods (reliability ratio = 1.08).

Table 1: Cross-Lagged Correlations between Mathematical Aptitude and Mathematical Problem-Solving Performance at Time-1 to Time-3

(N=480)

Time-1 to time-2 Cross-lags	R	z	p	Time-2 to time-3 Cross-lags	r	z	p	Time-1 to time-3 Cross-lags	r	z	p
MA_1MP_2	.49	-1.82	NS	MA_2MP_3	.51	-1.80	NS	MA_1MP_3	.57	-1.80	NS
MP_1MA_2	.56			MP_2MA_3	.63			MP_1MA_3	.63		

z is based on Pearson Filon. If z e" 1.96, difference in cross-lags at 0.05 level (two-tailed)

The statistical values as shown in the Table 1 reveal that the obtained corrected cross-lagged correlations between mathematical aptitude at time-1 and mathematical problem-solving performance at time-2 ($r_{MA_1MP_2} = .49$) and mathematical aptitude at time-2 and mathematical problem-solving performance at time-1 ($r_{MP_1MA_2} = .56$) was found to be significant. The obtained Pearson-Filon z value (-1.82) of the difference between $r_{MA_1MP_2}$ and $r_{MP_1MA_2}$ (.56 - .49 = .07) was found to be not significant. Similarly, the obtained Pearson-Filon z value (-1.80) of the difference between $r_{MA_2MP_3}$ (.51) and $r_{MP_2MA_3}$ (.63) was found to be not significant. It can also be seen from the Table-1 that the obtained cross-lagged correlation between mathematical aptitude at time-1 and mathematical problem-solving performance at time-3 ($r_{MA_1MP_3} = .57$) and mathematical problem-solving performance at time-1 and mathematical aptitude at time - 3 ($r_{MP_1MA_3} = .63$) were found to be significant. The Pearson-Filon z value (-1.80) of the difference between $r_{MA_1MP_3}$ and $r_{MP_1MA_3}$ was found to be not significant. The analysis indicated that significant causal relationship was not to be found at .05 level of significance. But at .10 level of significance the analysis revealed a symmetric relationship between mathematical aptitude and mathematical problem-solving performance i.e., both are mutually reinforcing each other. Statistical value 1.95 may not be significant at .05 levels (two tailed) but meta-analysis considers the magnitude of effect size in generalizing the findings (Tyagi & Singh, 2016). For example, the difference in mean mathematical creativity scores of two groups (M_1 = 12.0, M_2 = 12.00001) is negligible and this difference may be due to a lack of measurement precision, but if N is sufficiently large and data satisfy the parametric conditions, this small difference may be significant and one can generalize that one group is significantly better than the other. Thus the null hypothesis between two cross-lagged correlations is spurious, can be rejected.

As can be seen from the Figure-2, autocorrelation for mathematical aptitude ($r_{MA\,MA}$, $r_{MA\,MA}$, and $r_{MA\,MA}$) and mathematical problem-solving performance ($r_{MP_1MP_2}$, $r_{MP_2MP_3}$, and $r_{MP_1MP_3}$) were found to be ranged from .58 to .67 and .71 to .78 respectively and significant at .05 level. These values show the stability and reliabilitv of both the tests. Figure 2 further shows that synchronous correlations ($r_{MA_1MP_1}$, $r_{MA_2MP_2}$ and $r_{MA_3MP_3}$) were found to be ranged from .54 to .66. These significant values reveal a priori relationship between mathematical aptitude and mathematical problem-solving performance. But an abnormal change in the synchronous correlations with time-1 to time-3 may indicate the possible influence of third variables.

Discussion

The present study involved a tentative attempt to ferret out and focus upon the presence of causal relationship between mathematical aptitude and mathematical problem-solving performance. Using cross-lagged panel analytic procedure with three-wave and two-variable (3w2v) the findings of the study revealed a symmetric relationship between mathematical aptitude and mathematical problem-solving performance i.e., both are reinforcing to each other equally. The most probable inference is that the interval chosen between time 1 and time 2 is not appropriate one which captures the dynamics of the causal relationship. An abnormal change in the synchronous correlations may indicate the possible influence of third variables (Tyagi & Wotruba, 1993). The investigator can never rule out the existence of third variables that underlie the cross-lag correlations between these two constructs.

One of the strengths of the current study is that symmetric relationship was investigated with three measurement phases. The findings of the study should be viewed as one step forward the identification of relationship between mathematical aptitude and mathematical problem-solving performance as well as one step prior to a true experimental study of the causal relationship between the two variables. However, CLPC procedure may be one of the most appropriate designs in which the variables are not typically subject to experimental manipulation or random assignment of participants is not possible. These results show little chance of unidirectional causality between these two constructs. The values may be affected due to post hoc fallacy i.e., due to effect of third variables which were not consider in the study. Hence the results of this study may be show only a mutually reinforcing (symmetric) relationship rather than causal. The present study considered a relatively small sample and short lag interval. Although a critical transition time of investigation was covered by the investigator, nevertheless, a longer time and large sample size may help to present more clearer the unidirectional causal relationship between mathematical aptitude and mathematical problem-solving performance.

Notwithstanding, the findings of the present study have important implications for theory and practice in education. Aptitude is the potential of a person to learn effectively in future engagement, it is therefore, important that schools should develop curriculum and use of teaching methods that are effective in developing skills of students in numerical and verbal domains as a means of improving performance of students. Mathematical aptitude and mathematical problem-solving performance both are commonly identified as important areas for the students' growth in the school curriculum. Longitudinal studies should be conducted to investigate the causal relationship between these two constructs. Future research on this topic might consider other variables such as mathematical intelligence, self-concept in mathematics and mathematical creativity that may have a role to play in any such causal relationship. Past and present studies have not addressed these concerns in details. In future, cross-lag relations may be tested with different time periods (four lags) with large sample size between measurements. It may be that shorter or longer time gaps between two measurements may lead high or low cross-lagged relationships. Future longitudinal and experimental research should be conducted to explicitly investigate the relationship between mathematical aptitude and mathematical problem-solving performance.

Acknowledgement: Special thanks to Prof. Bhoodev Singh in the Education Department at BHU Varanasi, India for suggestion to contents of this document.

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